第十九届中国图象图形学学会青年科学家会议

Title : Rank Consistency Induced Multi-view Subspace Clustering via Low-rank Matrix Factorization

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Abstract

- This paper proposes a rank consistency induced multi-view subspace clustering model to pursue a consistent low-rank structure among view-specific self-expressiveness coefficient matrices.
- To facilitate a practical model, we parameterize the low-rank structure on all self-expressiveness coefficient matrices through the tri-factorization along with orthogonal constraints. This specification ensures that self-expressiveness coefficient matrices of different views have the same rank to effectively promote the structural consistency across multi-views, which can learn a consistent subspace structure and fully exploit the complementary information.
- An efficient algorithm with guaranteed convergence is proposed to solve the formulated optimization problem. Extensive experiments on several benchmark datasets demonstrate the superiority and effectiveness of the proposed method.

Introduction

- Given a set of data vectors $X = [x_1, x_2, \cdots, x_n] \in R^{d\times n}$ drawn from a union of k subspaces $\{S_i\}_{i=1}^{k}$. The task of subspace clustering is to segment the data into several disjoint clusters according to the underlying subspaces they are drawn from.
- Spectral Clustering (SC) is a common framework for subspace clustering, which aims to learn a "good" affinity matrix.
- Low-Rank Representation (LRR) (G. Liu et al., TPAMI 2013):
$$min_Z \ \|Z\|_* \quad s.t.\ X = XZ$$
- Given multi-view data $\{X_1, X_2, \cdots, X_V\}$, the general model of Multi-view Subspace Clustering (MSC) (J. Guo et al., TPAMI 2023):
$$min_{Z_v \in \mathbb{C}} \sum_{v=1}^{V} \|X_v - X_v Z_v\|_l + \lambda\Omega(Z_1, Z_2, \cdots, Z_v)$$
- There are two important principles for multi-view learning: complementarity and consistency.

Key Design

A. Rank Consistency Induced Multi-view Subspace Clustering (RC-MSC) (J. Guo et al., TNNLS, 2022)

1) **Complementarity:** extending LRR into multi-view learning
$$min_{Z_v,E_v} \sum_{v=1}^{V} \frac{\lambda}{V}\|Z_v\|_* + \|E_v\|_{2,1} \quad s.t. X_v = X_v Z_v + E_v.$$
2) **Consistency:** rank consistency structural constraint
$$rank(Z_1) = rank(Z_2) = \cdots = rank(Z_V) \le K, K \ll n.$$

The formulation of RC-MSC:
$$min_{Z_v,E_v} \sum_{v=1}^{V} \lambda/V\|Z_v\|_* + \|E_v\|_{2,1}$$
$$s.t.\ \ X_v = X_v Z_v + E_v, rank(Z_1) = \cdots = rank(Z_V) \le K.$$

B. RC-MSC via Matrix Factorization

Parameterize the low-rank coefficient Z_v with tri-factorization along with orthogonal constraints and shared core matrix C :
$$Z_v = L_v C R_v^T,$$
$$L_v^T L_v = R_v^T R_v = I_K, L_v, R_v \in R^{n\times K}, C \in R^{K\times K}.$$

Proposition: Given a matrix Z, which can be decomposed as LCR^T, where $L, R \in R^{n\times K}$, $C \in R^{K\times K}$, and $L^T L = R^T R = I_K$. Then, Z and C have the identical singular values. Thus, the rank of Z equals to the rank of C, i.e., $rank(Z) = rank(C) \le K$, $\|Z_v\|_* = \|C\|_*$.

Remark: $rank(Z_v) = rank(C) \le K, \|Z_v\|_* = \|C\|_*$.

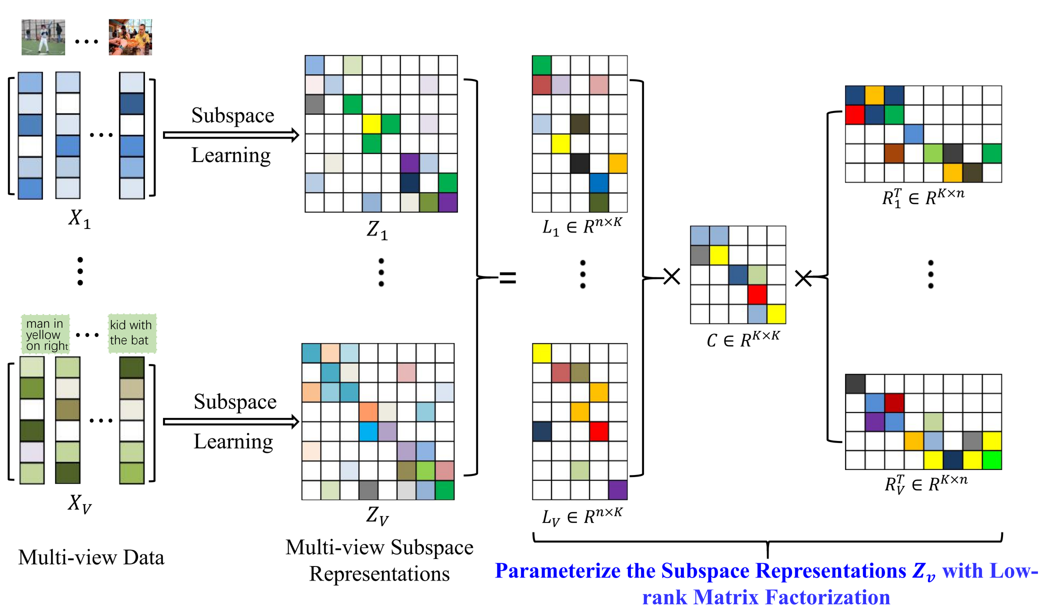

Fig 1. The framework of the proposed RC-MSC.

C. The final objective function of RC-MSC

$$min_{Z_v,E_v} \lambda\|C\|_* + \sum_{v=1}^{V} \|E_v\|_{2,1}$$
$$s.t. X_v = X_v Z_v + E_v, Z_v = L_v C R_v^T, L_v^T L_v = R_v^T R_v = I_K.$$

All views share the similar underlying clustering structure to achieve structure agreement and representation complementarity.

Solution

A. Optimization with Alternating Direction Method of Multipliers

- Partial augmented Lagrangian function
$$\mathcal{L} = \lambda\|C\|_* + \sum_{v=1}^{V} \left(\|E_v\|_{2,1} + \Phi(Q_v^1, X_v - X_v Z_v - E_v)\right) + \sum_{v=1}^{V} \Phi(Q_v^2, Z_v - L_v C R_v^T),$$

where Q_v^1 and Q_v^2 are dual variables, $\Phi(A, B) = \langle A, B\rangle + \mu\|B\|_F^2/2$.

- Alternating Direction Method of Multipliers (ADMM)

1. Update L_v: $min_{L_v} \Phi(Q_v^2, Z_v - L_v C R_v^T)$ $s.t.$ $L_v^T L_v = I_K$;
2. Update R_v: $min_{R_v} \Phi(Q_v^2, Z_v - L_v C R_v^T)$ $s.t.$ $R_v^T R_v = I_K$;
3. Update C: $min_C \lambda\|C\|_* + \sum_{v=1}^{V} \Phi(Q_v^2, Z_v - L_v C R_v^T)$;
4. Update Z_v: $min_{Z_v} \Phi(Q_v^1, X_v - X_v Z_v - E_v) + \Phi(Q_v^2, Z_v - L_v C R_v^T)$;
5. Update E_v: $min_{Z_v} \|E_v\|_{2,1} + \Phi(Q_v^1, X_v - X_v Z_v - E_v)$;
6. $Q_v^1 = Q_v^1 + \mu(X_v - X_v Z_v - E_v)$, $Q_v^2 = Q_v^2 + \mu(Z_v - L_v C R_v^T)$;
7. $\mu = \min(\rho\mu, \mu_{max})$, ρ>1.

B. Convergence Results

Theorem. Let $\Upsilon^t = \{Z_v^t, E_v^t, L_v^t, R_v^t, C^t, Q_v^{1,t}, Q_v^{2,t}\}$ be the generated sequences of ADMM, assume that $\lim_{t\to\infty} \mu^t(Z_v^{t+1} - Z_v^{t+1}) = 0$, $\lim_{t\to\infty} \mu^t(E_v^{t+1} - E_v^{t+1}) = 0$, then the sequence Υ^t satisfies:

1) The sequence Υ^t is bounded;
2) The sequence Υ^t has at least one accumulation point. And, any accumulation point is a stationary point of optimization problem.

Performance

- Construct the fused affinity matrix $S = 1/V(\sum_{v=1}^{V} |Z_v| + |Z_v^T|)$.
- Segment the data into k groups by Normalized Cuts.

	Yale		UCI-Digits		BBCSport		Caltech101-7	
	ACC	NMI	ACC	NMI	ACC	NMI	ACC	NMI
LTMSC	73.88	75.81	89.33	82.20	46.72	18.14	85.35	56.39
LMSC	60.09	63.61	78.86	75.20	92.10	84.61	86.64	57.23
CSMSC	78.70	80.28	90.77	83.77	94.85	85.56	86.82	59.27
FMRSC	81.64	79.31	85.11	77.03	87.13	80.69	84.46	49.36
RC-MSC	**83.94**	**84.21**	**92.15**	**84.84**	**96.69**	**89.59**	**87.90**	**60.01**

Table 1: Clustering results on the four benchmark datasets.

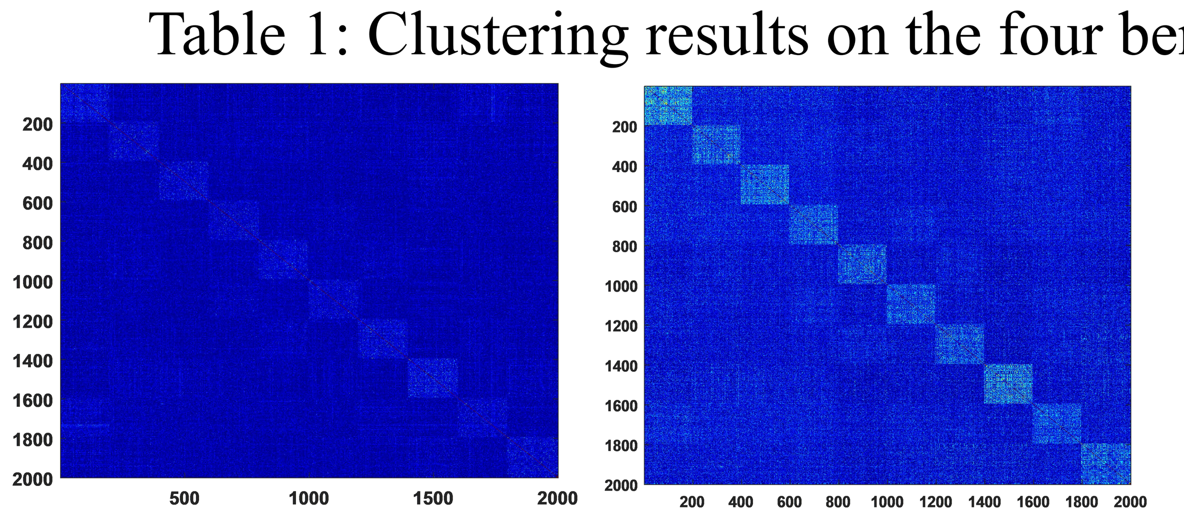

Fig 2. Affinity matrices obtained by CSMSC and RC-MSC on the UCI-Digits dataset.

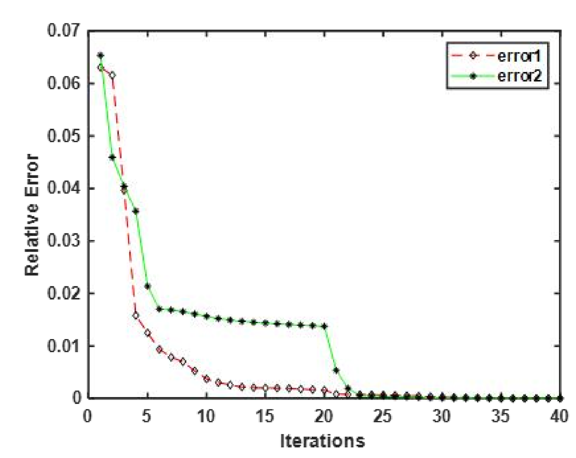

Fig 3. Convergence curve of RC-MSC.

Main Reference

[1]G. Liu, Z. Lin, et al. Robust recovery of subspace structures by low-rank representation [J]. IEEE T-PAMI, 2013, 35(1): 171-184.
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[3]**J. Guo**, Y. Sun, et al. Logarithmic Schatten-p Norm Minimization for Tensorial Multi-view Subspace Clustering [J]. IEEE T-PAMI, 2023, 45(3), 3396-3410.